Dependence of Steady Mach Reflections on the Reflecting-Wedge Trailing-Edge Angle

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Introduction

AS indicated in Ref. 1, two shock-wave-reflection configurations are possible in steady flows, regular reflection (RR) and Mach reflection (MR). The RR consists of two shock waves: the incident shock wave i and the reflected shock wave r. They meet at the reflection point R, which is located on the reflecting surface. The flow states are (0) ahead of i, (1) behind i, and (2) behind r. The angle of incidence ϕ_1 of an RR is sufficiently small so that the streamline deflection θ_1 caused by the incident shock wave i can be canceled by the opposite streamline deflection θ_2 caused by the reflected shock wave r. Therefore, the boundary condition of an RR is $\theta_1 - \theta_2 = 0$. The MR consists of three shock waves: the incident shock wave i, the reflected shock wave r, and the Mach stem m, and one slipstream s. They all meet at a single point, the triple point T. The Mach stem m is usually a curved shock wave, which is perpendicular to the line of symmetry at the reflection point R. The flow states are (0) ahead of i and m, (1) behind i, (2) behind r, and (3) behind m. Unlike the case of an RR where the net deflection of the streamline is zero, in the case of an MR the net deflection of the streamline is nonzero, in general, and the streamlines behind the triple point are directed toward the line of symmetry. Because the streamlines on both sides of the slipstream must be parallel, the boundary condition of an MR is $\theta_1 - \theta_2 = \theta_3$.

Note here that the aforementioned boundary conditions are based on local considerations in the vicinities of the reflection point of an RR and the triple point of an MR. For these conditions to be global, the discontinuities, i.e., shock waves and slipstream, must be straight so that the flow regions bounded by them are uniform.

As mentioned in Ref. 1 (Chap. 3.3), one of the unsolved problems, until a few years ago, associated with steady MR wave configurations was the mechanism determining the height of the Mach stem. Consider Fig. 1, where the solid lines describe the four discontinuities of a steady MR wave configuration with the triple point T, shock waves i, r, and m, and slipstream s. If one selects any point along the incident shock wave i and draws from it three lines parallel, respectively, to the reflected shock wave r, to the Mach stem m, and to the slipstream s, then there is a new triple point T^*, with its four discontinuities. The two triple points T and T^*, as well as all of the other triple points that could have been obtained by choosing different locations for T^* along the incident shock wave i, completely satisfy the governing equations of the three-shock theory, which is the analytical model describing MR wave configurations. However, if an experiment with the same initial conditions is repeated, then only one wave configuration out of the infinity of possible ones is always obtained.

The first analytical model for predicting the Mach stem height of a steady MR was presented in Ref. 2. In their model (Fig. 2), they assumed that 1) the Mach stem, the slipstream, and the line of symmetry form a one-dimensional converging nozzle; 2) the throat of this nozzle is at the point where the leading characteristic of the expansion fan emanating from the trailing edge of the reflecting wedge intersects the slipstream (point E in Fig. 2); and 3) the flow in region (3) is isentropic and reaches sonic conditions at the throat.

The predicted values of the height of the Mach stem for $M_0 = 4.96$, as obtained using this model, are shown with a dashed line in Fig. 3 together with the experimental results of Ref. 3. The agreement between the predicted values and the experimental results is not satisfactory enough.

In an attempt to improve the analytical predictions, in Refs. 4 and 5 it was argued that unlike Ref. 2, which considered only the leading characteristic of the expansion fan, the entire expansion fan should be considered. Their argument was based on the fact that the interaction between the expansion fan and the slipstream cannot result in a throat at point E (Fig. 2), where the head of the expansion fan intersects the slipstream, but farther downstream, e.g., at point C, as shown in Fig. 2. Thus, unlike the model considered in Ref. 2, where the angle of the trailing wedge is not important because the head of the expansion fan is determined by the flow Mach number upstream of it, in the models proposed in Refs. 4 and 5 the angle of the trailing wedge is important as it affects the entire expansion fan.

Unfortunately, the two analytical models developed in Refs. 4 and 5 led to contradicting results as far as the height of the Mach stem was concerned. Whereas the model developed in Ref. 4 predicted a strong dependence on the wedge angle of the trailing edge (see the solid lines in Fig. 3), the model developed in Ref. 5 did not reveal any dependence. Consequently, it was decided to clear this issue by conducting simultaneously experimental and numerical investigations.

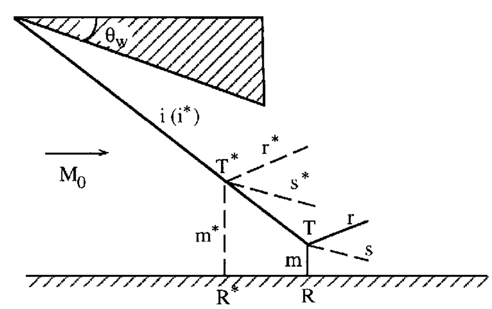

Fig. 1 Schematic illustration of two different MR wave configurations for identical conditions that satisfy the three-shock theory.

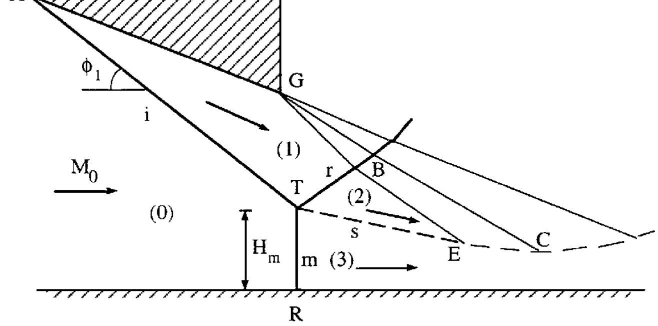

Fig. 2 Schematic illustration of an MR wave configuration including the interaction of the expansion fan, emanating from the trailing edge of the reflecting wedge, with the slipstream.

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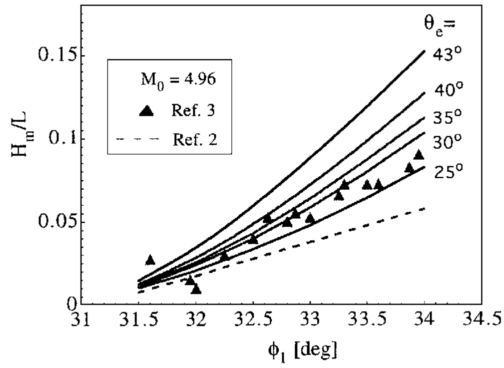

Fig. 3 Dependence of the nondimensional Mach stem height on the angle of the trailing edge of the reflecting wedge; comparison between analytical predictions and experimental results.

Present Study

Two simultaneous numerical investigations, using different codes, were conducted. The numerical method, in the first study, was the LCPFCT algorithm of Ref. 6, which is a one-dimensional flux corrected transport algorithm, for solving generalized continuity equations, with fourth-order phase accuracy and minimum residual diffusion. Details regarding the way by which the code was used for simulating steady MRs are given in Ref. 7. The simulations were carried out with $M_0 = 4.96$, $\theta_w = 26.56$ deg and five different trailing-edge wedge-angles, $\theta_e = 0$, 63.44, 126.88, 135, and 153.44 deg. The results did not reveal any dependence of the Mach stem height on the wedge angle of the trailing edge of the reflecting wedge.

A computer code based on a W-modified Godunov's scheme was developed and used in the second study. It is an explicit monotone scheme of second-order accuracy in space and time.[8] The scheme uses additive corrections to fluxes in the governing equations, with subsequent employment of the first-order accurate Godunov scheme to the modified system of equations. The method was realized on a W-stencil with orientation depending on the local flow velocity. The method could be viewed as a combination of van Leer's[9] and Harten's[10] schemes. In the calculations of the shock wave reflection the W-modification of Godunov's scheme was used with a moving adaptive curvilinear grid. The stationary solutions were calculated using relaxation techniques. The simulations with $M_0 = 4.96$, $\theta_w = 27.46$ deg, and $\theta_e = 122.54$, 127.54, 132.54, 137.54, 142.54, 147.54, 152.54, and 157.54 deg are shown in Figs. 4a–4h. They clearly show that the Mach stem height does not depend on the wedge angle of the trailing edge.

To complement these numerical studies, an experimental study was also conducted. Details of the experimental facility and the setup are given in Ref. 11. Seven experiments with $M_0 = 4.96$, $\theta_w = 32$ deg, and $\theta_e = 45$, 60, 90, 100, 110, 120, and 130 deg were conducted. The results clearly indicated that the Mach stem height was not affected by the change in the wedge angle of the trailing edge of the reflecting wedge.

Discussion and Conclusions

The foregoing numerical and experimental studies clearly indicated that the wedge angle of the trailing edge of the reflecting wedge does not have any noticeable effect of the location of the triple point of the MR and, hence, the height of its Mach stem. These results are in full agreement with predictions based on the analytical model presented in Ref. 5 and in contradiction with the results of the analytical model of Ref. 4.

However, based on the fact that the experimental results clearly indicated that the throat of the nozzle, formed by the slipstream and the line of symmetry, was not at the point where the leading characteristic of the expansion fan intersected the slipstream but farther downstream, one must conclude that the basic assumptions of the analytical model of Ref. 2 are wrong, i.e., the location of the throat is not at point E (Fig. 2) and the flow behind the Mach stem does not reach sonic conditions, not even at the actual throat that is formed farther downstream. Had the flow reached sonic conditions

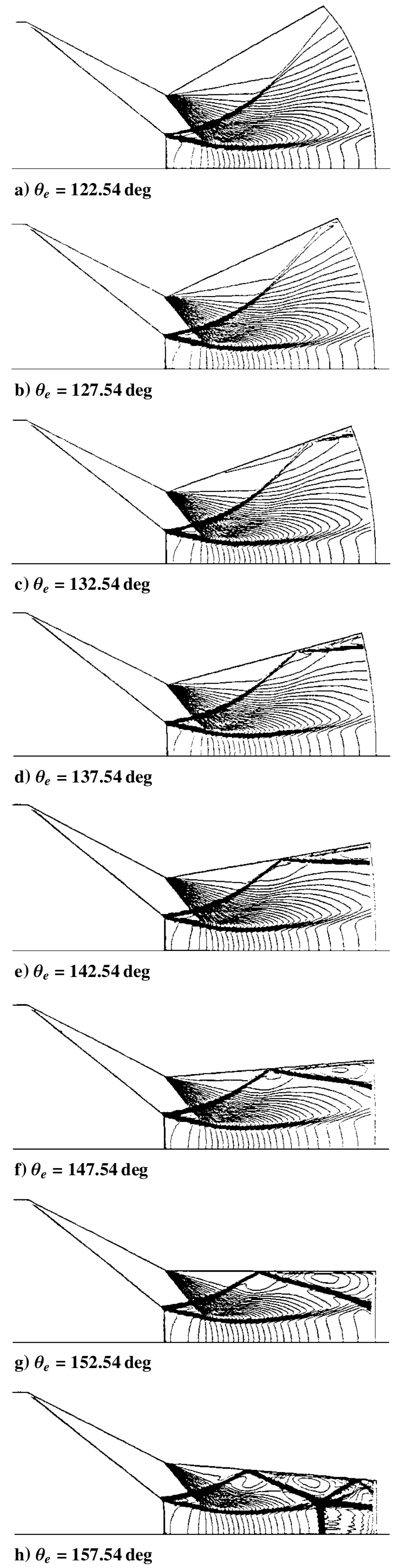

Fig. 4 Dependence of the Mach stem height on the angle of trailing edge of the reflecting wedge; numerical results.

at the actual throat, whose location is governed by the wedge angle of the trailing edge, a clear dependence on this angle must have existed.

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Stabilization of Supersonic Combustion by a Free Recirculating Bubble: A Numerical Study

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I. Introduction

DURING the past few years, the problem of ignition and stabilization of combustion in supersonic flows has been addressed by various authors, from both experimental[1] and numerical[2] points of view. An issue of fundamental importance is the stabilization of combustion when the vehicle flight Mach number is low. In this case, the flow Mach number and static temperature within the combustion chamber are too low to allow self-ignition in mixing layers and jets, and stable oblique detonation regimes are difficult to obtain. One way to circumvent this shortcoming, previously suggested by Winterfeld[3] and by Nedungadi and Lewis,[4] is the stabilization of combustion within a free recirculating bubble. The problem of a free recirculation bubble in supersonic nonreactive flows has been previously studied by various authors.[4–10] In these works, a recirculation bubble is generated by the interaction of an oblique shock wave and an incoming flow containing a finite region where a total pressure deficit exists. This total pressure deficit region is, in the aforementioned papers, a region of swirling flow, generated either by a wing-like structure or an injector. Depending on the relative intensities of the swirl and of the shock wave, it is possible to obtain either a small bulging of the shock or a large deformation of the shock wave, the latter leading to the formation of a free recirculating bubble. More recently, Mahesh[10] has shown that the bubble can be obtained without any swirl, the controlling parameter in the formation of the free recirculating bubble being the velocity deficit.

In the present work, the use of a recirculation bubble is proposed to achieve full premixing, ignition, and stabilized combustion in supersonic flows. A first description of the flowfield obtained when combustion occurs is given.

II. Mathematical Formulation of the Problem and Numerical Procedure

In this Note, the two-dimensional unsteady conservation equations of mass, momentum, energy, and species mass fraction are solved for a multicomponent gas mixture[2] in the geometry shown in Fig. 1 of a uniform airflow surrounding a methane axisymmetric jet. The turbulent transport model used is a gradient closure, with a mixing length turbulent viscosity $\nu_t = C_\delta l^2 |\partial u/\partial y|$, where $C_\delta = 0.008$ and l is the mixing length, assumed to be equal to the scale δy of the gradient, and δU is taken as the maximum difference of velocity in the incoming flow, $400 \le \delta U \le 700$ m/s. In fact, this leads to what can be considered an overestimation of the turbulent viscosity $\nu_t = 4000$ cm^2/s. This relatively large value of ν_t can be justified by the existence of large-scale fluctuations in the flow external to the configuration studied. Computations showing the exact influence of the turbulent viscosity are currently in progress. The reaction rate of methane and air is calculated assuming a global chemical kinetics model[11] and using the average temperature and mass fractions. This global modeling of the chemical process is known to not be very accurate for computing the induction time for a large range of temperature and pressures. Even if the drastic modeling of both turbulent transports and chemistry used is not suited to predict in detail the local characteristics of the flow, it is deemed sufficient to describe the gross features of the phenomena studied in this Note.

The numerical procedure used here is based on the Liou[12] advection upstream splitting method, with a second-order MUSCL extrapolation and MINMOD limiter for the primitive variables (ρ, u, v, p, Y_k). Details concerning the solution procedure can be found in Ref. 2.

III. Results and Discussion

The interaction of a swirling flow with an oblique shock wave may lead to complex three-dimensional phenomena.[4–9] To retain only the most significant aspects of such an interaction, we treat this problem here in a two-dimensional axisymmetric form. In a way that is similar to that of the work of Metwally et al.[8] and Mahesh,[10] but using a different flow/shock wave interaction, we obtain free recirculating bubbles without swirl when the velocity of the core is smaller than the external flow velocity.

As shown schematically in Fig. 1, a central stream of partially premixed fuel (stoichiometric methane/air mixture, denoted by the subscript c) that is surrounded by an airstream (denoted by the subscript ∞) is considered. Both streams have the same static pressure $p_\infty = p_c$ and, for the sake of simplicity, total temperature $T_{t\infty} = T_{tc}$ but different velocities, i.e., u_∞ and u_c. At $x = 0$ both

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